

# Emergence and oscillation of cosmic space by joining M1-branes

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**Abstract** Recently, it has been proposed by Padmanabhan that the difference between the number of degrees of freedom on the boundary surface and the number of degrees of freedom in a bulk region leads to the expansion of the universe. Now, a natural question arises; how could this model explain the oscillation of the universe between contraction and expansion branches? We try to address this issue in the framework of a Bionic system. In this model,  $M0$ -branes join to each other and give rise to a pair of  $M1$ -anti- $M1$ -branes. The fields which live on these branes play the roles of massive gravitons that cause the emergence of a wormhole between them and formation of a BIon system. This wormhole dissolves into  $M1$ -branes and causes a divergence between the number of degrees of freedom on the boundary surface of  $M1$  and the bulk leading to an expansion of  $M1$ -branes. When  $M1$ -branes become close to each other, the square energy of their system becomes negative and some tachyonic states emerge. To remove these states,  $M1$ -branes become compact, the sign of compacted gravity changes, causing anti-gravity to arise: in this case, branes get away from each other. By articulating  $M1$ -BIons, an  $M3$ -brane and an anti- $M3$ -brane are created and connected by three wormholes forming an  $M3$ -BION. This new system behaves like the initial system and by closing branes to each other, they become compact and, by getting away from each other, they open. Our universe is located on one of these  $M3$ -branes and, by compactifying the  $M3$ -brane, it contracts and, by opening it, it expands.

## 1 Introduction

The origin of the universe expansion has been described recently by Padmanabhan [1]. It has been proposed that the expansion of the universe happens as a result of a deviation between the surface degrees of freedom on the holographic horizon and the bulk degrees of freedom [1]. To date, several papers investigated this interesting proposal and its implications for cosmology [2–8]. For example, the Padmanabhan proposal has been used to deduce the Friedmann equations of an  $(n + 1)$ -dimensional Friedmann–Robertson–Walker (FRW) universe in the framework of general relativity, Gauss–Bonnet gravity and Lovelock gravity [2]. In another case, the proposal has been extended to brane cosmology, scalar-tensor cosmology and  $f(R)$  gravity [3]. In another scenario, with the help of Padmanabhan’s proposal, the Friedmann equations have been derived of the universe in higher dimensional space-time in different gravities like Gauss–Bonnet and Lovelock gravity with general spatial curvature [4]. In another investigation, the Padmanabhan idea has been generalized to the non-flat universe corresponding to the spatial curvature parameter  $k = \pm 1$  [5, 6]. Besides, in [7], the Padmanabhan proposal has been investigated in the context of the generalized uncertainty principle (GUP). And in a recent work, the Padmanabhan model has been considered in a Bionic system and it has been argued that the difference between the degrees of freedom inside and outside the universe is due to the evolutions of branes in extra dimensions [8]. In general, the BION is a configuration of two branes which are connected by a wormhole [9–13].

On the other hand, recent investigations show that the universe may oscillate between contraction and expansion

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branches [14–16]. A naturally arising question is whether the Padmanabhan proposal could explain the universe's oscillation. We try to answer this question in the framework of a Bionic system. In previous studies, it has been argued that the Big Bang may be removed in string theory and replaced by  $N$  fundamental strings [13]. In this model, first,  $N$  fundamental strings transit to  $N$  pairs of  $M0$ -brane and anti- $M0$ -brane. Then these branes glue to each other and build up a Bionic system which is a configuration of  $M3$ -brane and an anti- $M3$ -brane in addition to a wormhole. Our universe is located on one of these  $M3$ -branes and interacts with the other universe via the wormhole [13].

In this paper, we will extend those calculations and show that by joining  $M0$ -branes, a pair of  $M1$ -anti- $M1$ -branes could be constructed. At that stage, two types of fields are produced and interact with branes. One type plays the role of the scalar field in transverse dimensions and the other one appears as graviton field on the  $M1$ -branes. These gravitons lead to the emergence of a wormhole between branes and hence the formation of a BIon system. The evolution of the BIon leads to the difference between the number of degrees of freedom on the boundary surface of  $M1$  and the bulk; and this difference is the main cause of the expansion of  $M1$ -branes in the Padmanabhan picture. When  $M1$ -branes approach each other, the square energy of the branes' system becomes negative and the system transits to the tachyon phase. To remove these tachyon states,  $M1$ -branes become compact and gravity turns to anti-gravity. Under that conditions, the branes get away from each other and begin to be opened. These BIONS glue each other and form a bigger BIon, which includes an  $M3$ -brane and an anti- $M3$ -brane in addition to three wormholes connecting them. The  $M3$ -branes oscillate between compact and open branches due to the oscillation of the initial  $M1$ -branes. Our universe is placed on one of these  $M3$ -branes. By compactifying the  $M3$ -branes, it contracts and by opening the  $M3$ -branes, it expands.

The outline of the paper is the following. In Sect. 2, we consider the formation and the expansion of  $M1$ -branes. We also study the process of the formation of  $M3$  from  $M1$ -branes and obtain the difference between the number of degrees of freedom of the universe in terms of BIon evolution. In Sect. 3, we discuss how, by compactifying branes, gravity turns to anti-gravity and the contraction of the branches begins. The last section is devoted to a discussion and to our conclusions.

## 2 Cosmic expansion in Padmanabhan model

In previous studies, it has been shown that by joining  $M0$ -branes to each other, a pair of  $M1$ -anti- $M1$ -branes can be formed [13]. The fields on these branes play the role of gravi-

tons and cause the formation of a wormhole between these branes. These graviton fields are the main cause of the difference between the number of degrees of freedom of brane and bulk and hence cause an expansion. By closing  $M1$ -branes, they bend, become compact, and gravity turns to anti-gravity. By gluing  $M1$ -branes,  $M3$ -branes are formed and our universe is located on one of them. These branes expand and become compact like the initial  $M1$ -branes and this fact leads to an oscillation of our universe between expansion and contraction branches. The action of  $M1$  can be written as [13, 17–28]:

$$S = -T_{M1} \int d^2\sigma \, STr \sqrt{-\det(P_{abc}[E_{mnl} + E_{mij}(Q^{-1} - \delta)^{ijk}E_{klm}] + \lambda F_{abc})\det(Q^i_{j,k})}$$

where

$$E_{mnl}^{\alpha,\beta,\gamma} = G_{mnl}^{\alpha,\beta,\gamma} + B_{mnl}^{\alpha,\beta,\gamma}, \quad Q^i_{j,k} = \delta^i_{j,k} + i\lambda[X^j_\alpha T^\alpha, X^k_\beta T^\beta, X^l_\gamma T^\gamma]E^{\alpha,\beta,\gamma}_{k'jl}, \quad (1)$$

$$F_{abc} = \partial_a A_{bc} - \partial_b A_{ca} + \partial_c A_{ab}. \quad (2)$$

Here  $X^M = X^M_\alpha T^\alpha$ ,  $A_{ab}$  is a 2-form gauge field,

$$[T^\alpha, T^\beta, T^\gamma] = f^{\alpha\beta\gamma} T^\eta, \quad [X^M, X^N, X^L] = [X^M_\alpha T^\alpha, X^N_\beta T^\beta, X^L_\gamma T^\gamma] \quad (3)$$

where  $\lambda = 2\pi l_s^2$ ,  $G_{mnl} = g_{mn}\delta_{n,l}^{n'} + \partial_m X^i \partial_{n'} X^i \sum_j (X^j)^2 \delta_{n,l}^{n'} + \partial_{n'} \partial_m X^i \partial_m \partial_{n'} X^i \delta_{n,l}^{n'}$  and  $X^i$  are scalar fields of mass dimension. Here  $a, b = 0, 1, \dots, p$  are the world-volume indices of the  $Mp$ -branes,  $i, j, k = p+1, \dots, 9$  are indices of the transverse space, and  $m, n$  are the 11-dimensional space-time indices. Also,  $T_{Mp} = 1/(g_s(2\pi)^p l_s^{p+1})$  is the tension of the  $Mp$ -brane,  $l_s$  is the string length and  $g_s$  is the string coupling. In previous studies, it has been shown that this action can be obtained by summing over the actions of  $pM0$ -branes; it is given by [13]

$$S_{M0} = T_{M0} \times \int dt \, Tr \left( \Sigma_{M,N,L=0}^{10} \langle [X^M, X^N, X^L], [X^M, X^N, X^L] \rangle \right). \quad (4)$$

To obtain the action (1) from the action of  $M0$ , we should use the following mappings [13, 17–28]:

$$\begin{aligned} & \langle [X^a, X^i, X^j], [X^a, X^i, X^j] \rangle \\ &= \frac{1}{2} \varepsilon^{abc} \varepsilon^{abd} (\partial_c X^i_\alpha) (\partial_d X^j_\beta) \langle (T^\alpha, T^\beta) \rangle \sum_j (X^j)^2 \\ &= \frac{1}{2} \langle \partial_a X^i, \partial_a X^i \rangle \sum_j (X^j)^2 \\ & \langle [X^a, X^b, X^j], [X^{a'}, X^{b'}, X^j] \rangle \\ &= \frac{1}{2} \varepsilon^{abc} \varepsilon^{a'b'c} (\partial_a \partial_b X^i_\alpha) (\partial_{a'} \partial_{b'} X^i_\beta) \langle (T^\alpha, T^\beta) \rangle \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2} \langle \partial_a \partial_b X_\alpha^i, \partial_{a'} \partial_{b'} X_\alpha^i \rangle \\
&\langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle \\
&= (F_{\alpha\beta\gamma}^{abc})(F_{\alpha\beta\eta}^{abc}) (\langle [T^\alpha, T^\beta, T^\gamma], [T^\alpha, T^\beta, T^\eta] \rangle) \\
&= (F_{\alpha\beta\gamma}^{abc})(F_{\alpha\beta\eta}^{abc}) f_{\sigma\kappa}^{\alpha\beta\gamma} h^{\sigma\kappa} f_{\kappa}^{\alpha\beta\eta} \langle T^\gamma, T^\eta \rangle \\
&= (F_{\alpha\beta\gamma}^{abc})(F_{\alpha\beta\eta}^{abc}) \delta^{\kappa\sigma} \langle T^\gamma, T^\eta \rangle = \langle F^{abc}, F^{abc} \rangle, \\
&i, j = p+1, \dots, 10 \quad a, b = 0, 1, \dots, p \quad m, n = 0, \dots, 10.
\end{aligned} \tag{5}$$

To obtain a similarity between branes and our real world, we assume that the 2-form fields play the role of gravitons and obtain the following results:

$$\begin{aligned}
A^{ab} &= g^{ab} = h^{ab} + \eta^{ab} \quad \text{and} \quad a, b, c = \mu, \nu, \lambda \\
\Rightarrow F_{abc} &= \partial_a A_{bc} - \partial_b A_{ca} + \partial_c A_{ab} \\
&= 2(\partial_\mu g_{\nu\lambda} + \partial_\nu g_{\mu\lambda} - \partial_\lambda g_{\mu\nu}) = 2\Gamma_{\mu\nu\lambda} \\
\langle F^\rho \sigma \lambda, F^\lambda \mu \nu \rangle &= \langle [X^\rho, X_\sigma, X_\lambda], [X^\lambda, X_\mu, X_\nu] \rangle \\
&= [X_\nu, [X^\rho, X_\sigma, X_\mu]] - [X_\mu, [X^\rho, X_\sigma, X_\nu]] \\
&\quad + [X^\rho, X_\lambda, X_\nu][X^\lambda, X_\sigma, X_\mu] \\
&\quad - [X^\rho, X_\lambda, X_\mu][X^\lambda, X_\sigma, X_\nu] \\
&= \partial_\nu \Gamma_{\sigma\mu}^\rho - \partial_\mu \Gamma_{\sigma\nu}^\rho + \Gamma_{\lambda\nu}^\rho \Gamma_{\sigma\mu}^\lambda - \Gamma_{\lambda\mu}^\rho \Gamma_{\sigma\nu}^\lambda \\
&= R_{\sigma\mu\nu}^\rho
\end{aligned}$$

and

$$\begin{aligned}
\kappa_\nu^\mu &= \delta_\nu^\mu - \sqrt{\delta_\nu^\mu - H_\nu^\mu} \\
H_{\mu\nu} &= g_{\mu\nu} - \eta_{mn} \partial_\mu X^m \partial_\nu X^n \\
H_{\mu\nu} &= h_{\mu\nu} + 2\Pi_{\mu\nu} - \eta^{\alpha\beta} \Pi_{\mu\alpha} \Pi_{\beta\nu} \\
X^m &= x^m - \eta^{m\mu} \partial_\mu \pi \\
\Pi_{\mu\nu} &= \partial_\mu \partial_\nu \pi
\end{aligned} \tag{6}$$

where  $\pi$  is the scalar mode and  $h^{ab}$  is the tensor mode of the graviton. As can be seen from the above equations, non-commutative relations between 2-form fields produce the exact form of the curvature tensor. Also, when scalars are attached to the branes, their index changes from  $i, j \rightarrow \mu\nu$  and they transit to the graviton mode. Previously, it has been shown that there are direct relations between  $\kappa$  and the curvature scalar ( $R$ ) [29–33]:

$$\delta_{\mu\nu}^{\rho\sigma} \kappa_\rho^\mu \kappa_\sigma^\nu = R. \tag{7}$$

Thus, gravity can easily be obtained from the non-commutative relations in  $M$ -theory. At this stage, we can derive the explicit form of the relevant action of  $M1$  in Eq. (1) in terms of gravity terms. We can write

$$\begin{aligned}
\det(Z) &= \delta_{b_1 b_2 \dots b_n}^{a_1 a_2 \dots a_n} Z_{a_1}^{b_1} \dots Z_{a_n}^{b_n} \quad a, b, c = \mu, \nu, \lambda, \\
Z_{abc} &= P_{abc}[E_{mnl} + E_{mij}(Q^{-1} - \delta)^{ijk} E_{klm}] + \lambda F_{abc},
\end{aligned}$$

$$\begin{aligned}
\det(Z) &= \det \left( P_{abc}[E_{mnl} + E_{mij}(Q^{-1} - \delta)^{ijk} E_{klm}] \right) \\
&\quad + \lambda^2 \det(F).
\end{aligned} \tag{8}$$

This equation helps us to derive the relevant terms of the determinant in the action (1) separately. Applying the relations in Eq. (7) in the determinants (8), we obtain

$$\det(F) = \delta_{\rho\sigma}^{\mu\nu} \langle F^{\rho\sigma} \lambda, F^\lambda \mu \nu \rangle = \delta_{\rho\sigma}^{\mu\nu} R_{\mu\nu}^{\rho\sigma}, \tag{9}$$

$$\begin{aligned}
&\det(P_{abc}[E_{mnl} + E_{mij}(Q^{-1} - \delta)^{ijk} E_{klm}]) \\
&= \delta_{\rho\sigma}^{\mu\nu} \left[ (g_\rho^\mu g_\sigma^\nu + g_\sigma^\nu \langle \partial^\mu X^i, \partial_\rho X^j \rangle \sum (X^i)^2 \right. \\
&\quad \left. + \langle \partial^\mu \partial^\nu X^i, \partial_\sigma \partial_\rho X^j \rangle + \dots) \right. \\
&\quad \left. - \frac{(g_\rho^\mu g_\sigma^\nu + g_\sigma^\nu \langle \partial^\mu X^i, \partial_\rho X^j \rangle \sum (X^i)^2 + \langle \partial^\mu \partial^\nu X^i, \partial_\sigma \partial_\rho X^j \rangle + \dots)}{[(\lambda)^2 \det([X_\alpha^j T^\alpha, X_\beta^k T^\beta, X_\gamma^{k'} T^\gamma])]} \right] \\
&= \delta_{\rho\sigma}^{\mu\nu} \left[ \kappa_\mu^\rho \kappa_\nu^\sigma \sum (X^i)^2 + (\partial_\lambda \kappa_\mu^\rho \partial^\lambda \kappa_\nu^\sigma) \right] \\
&\quad \times \left( 1 - \frac{1}{[(\lambda)^2 \det([X_\alpha^j T^\alpha, X_\beta^k T^\beta, X_\gamma^{k'} T^\gamma])]} \right) \\
&= \delta_{\rho\sigma}^{\mu\nu} \left[ \kappa_\mu^\rho \kappa_\nu^\sigma \sum (X^i)^2 + (\partial_\lambda \kappa_\mu^\rho \partial^\lambda \kappa_\nu^\sigma) \right] \left( 1 - \frac{1}{m_g^2} \right)
\end{aligned} \tag{10}$$

where  $m_g^2 = [(\lambda)^2 \det([X_\alpha^j T^\alpha, X_\beta^k T^\beta, X_\gamma^{k'} T^\gamma])]$  is the square of the graviton mass. It is clear that the graviton mass depends on the scalars which interact with the branes. This is because when the scalars collide with the branes, their index changes and they transit to the graviton. With this definition, we can calculate another term of the determinant:

$$\begin{aligned}
\det(Q) &\sim (i)^2 (\lambda)^2 \det([X_\alpha^j T^\alpha, X_\beta^k T^\beta, X_\gamma^{k'} T^\gamma]) \det(E) \sim \\
&\quad - [(\lambda)^2 \det([X_\alpha^j T^\alpha, X_\beta^k T^\beta, X_\gamma^{k'} T^\gamma])] \det(g) = m_g^2 \det(g).
\end{aligned} \tag{11}$$

By inserting Eqs. (7), (9), (10), and (11) into the action (1) and replacing  $\sum (X^i)^2 \rightarrow F(X)$ , we get

$$\begin{aligned}
S_{M1} &= -T_{M1} \int d^2 \sigma \left[ \sqrt{-g} \left( \delta_{\mu\nu}^{\rho\sigma} [\kappa_\mu^\rho \kappa_\nu^\sigma \sum (X^i)^2 + (\partial_\lambda \kappa_\mu^\rho \partial^\lambda \kappa_\nu^\sigma)] \right. \right. \\
&\quad \left. \left. - m_g^2 \delta_{\mu\nu}^{\rho\sigma} (R_{\rho\sigma}^{\mu\nu} + [\kappa_\mu^\rho \kappa_\nu^\sigma \sum (X^i)^2 + (\partial_\lambda \kappa_\mu^\rho \partial^\lambda \kappa_\nu^\sigma)]) \right) \right] \\
&= -T_{M1} \int d^2 \sigma \left[ \sqrt{-g} \left( F(X) R - m_g^2 \delta_{\mu\nu}^{\rho\sigma} R_{\rho\sigma}^{\mu\nu} - m_g^2 F(X) R \right. \right. \\
&\quad \left. \left. + \delta_{\rho\sigma}^{\mu\nu} (1 - m_g^2) (\partial_\lambda \kappa_\mu^\rho \partial^\lambda \kappa_\nu^\sigma) \right) \right].
\end{aligned} \tag{12}$$

Obviously, first order terms in nonlinear theories, like Lovelock and massive gravity, are present in this action. This means that there is a direct relation between  $M$ -theory and the effective theories of gravity. According to these calculations, there are two types of modes for the gravitons. We have scalar modes which are produced by attaching scalars to branes and tensor modes which are produced in the process of the formation of  $M1$  from  $M0$ -branes (see also [34,35]).

Using Eq. (1) and assuming the separation distance between two  $M1$  to be  $l_d$  and the length of each  $M1$  be  $l_1$ ,

we can obtain the relevant action for the interaction of an  $M1$  brane with an anti- $M1$ -brane:

$$\begin{aligned}
 A^{ab} &\rightarrow l_1 \quad X^2 \rightarrow l_d, \quad X^0 = t, \quad X^i = 0, \quad i \neq 0, 2, \\
 S &= -T_{M1} \int d^2\sigma \sqrt{l_d^2 + l_d^5} \\
 &\quad \times \sqrt{\left[l_1^2 + (l'_d)^2 + (l''_d)^2(1 + l_d^2)^{-1}\right] \left[1 - \frac{1}{l_d^3}\right] - (l'_1)^2(l_d)^2} \\
 &= -T_{M1} \int d^2\sigma V(l_d) \sqrt{D_{l_d, l_1}}, \\
 V(l_d) &= \sqrt{l_d^2 + l_d^5}, \\
 D_{l_d, l_1} &= \left[l_1^2 + (l'_d)^2 + (l''_d)^2(1 + l_d^2)^{-1}\right] \left[1 - \frac{1}{l_d^3}\right] - (l'_1)^2(l_d)^2
 \end{aligned} \quad (13)$$

where the prime denotes the derivative with respect to time. The equations of motion obtained from this action are

$$\begin{aligned}
 \left( \frac{(l'_d) \left[1 - \frac{1}{l_d^3}\right]}{\sqrt{D_{l_d, l_1}}} \right)' &= \frac{1}{\sqrt{D_{l_d, l_1}}} \left( 2l_d^{-4} [(l'_d)^2 \right. \\
 &\quad \left. + (l''_d)^2(1 + l_d^2)^{-1}] (l'_d) \right. \\
 &\quad \left. - 2(l'_1)^2(l_d)(l'_d) + \frac{V'}{V} \left[ D_{l_d, l_1} - (l'_d) \left[1 - \frac{1}{l_d^3}\right] \right] \right) \\
 \left( \frac{2(l'_1)(l_d)^2}{\sqrt{D_{l_d, l_1}}} \right)' &= \frac{1}{\sqrt{D_{l_d, l_1}}} \left( l_1 \left[1 - \frac{1}{l_d^3}\right] - \frac{V'}{V} [l'_1 l_d^2] \right).
 \end{aligned} \quad (14)$$

Solving these equations simultaneously, we obtain the approximate form of  $l_d$  and  $l_1$  in terms of the time:

$$\begin{aligned}
 l_1 &\sim \frac{l_{1,+}}{(t_s - t)^2} \left( \frac{t_s}{(t_s - t)} - 1 \right) e^{-l_0(t_s - t)}, \\
 l_d &\sim \frac{l_{d,0}}{t_s^{1/2}} (t_s - t)^{1/2} e^{\frac{-t}{t_s}},
 \end{aligned} \quad (15)$$

where  $t_s$  is time of collision between the  $M1$ -branes and  $l_{d,0}$  is the maximum distance between two  $M1$ -branes. To be sure that these solutions are true, specially near the point that branes collide to each other, we consider their correctness when the branes are close to each other ( $l_d \sim 0$ ). In this case, the size of two branes is very big ( $l_1 \sim \infty$ ) and the velocity of their motion and rate of their growth is large ( $l'_d \sim l'_1 \sim \infty$ ). For this state, the equations in (14) reduce to the following equations:

$$\begin{aligned}
 l_d &\sim 0 \quad l'_d \sim \infty \\
 &\Rightarrow \frac{V'}{V} D_{l_d, l_1} \sim \frac{l'_d(2l_d + 5l_d^4)}{l_d^2 + l_d^5}
 \end{aligned}$$

$$\begin{aligned}
 &\times \left( \left[ l_1^2 + (l'_d)^2 + (l''_d)^2(1 + l_d^2)^{-1} \right] \left[ 1 - \frac{1}{l_d^3} \right] - (l'_1)^2(l_d)^2 \right) \\
 &\sim 2l_d^{-4} [(l'_d)^2 + (l''_d)^2(1 + l_d^2)^{-1}] (l'_d) - 2(l'_1)^2(l_d)(l'_d) \\
 &\Rightarrow \left( \frac{(l'_d) \left[1 - \frac{1}{l_d^3}\right]}{\sqrt{D_{l_d, l_1}}} \right)' \sim \frac{1}{\sqrt{D_{l_d, l_1}}} \left( (l'_d) \left[1 - \frac{1}{l_d^3}\right] \right) \\
 &\Rightarrow X = \frac{1}{\sqrt{D_{l_d, l_1}}} \left( (l'_d) \left[1 - \frac{1}{l_d^3}\right] \right) \rightarrow \frac{dX}{dt} = \lambda X \rightarrow X = e^{\lambda t}, \\
 Y = \frac{1}{X} &\sim \frac{(l''_d)^2(1 + l_d^2)^{-1}}{(l'_d)^2 \left[1 - \frac{1}{l_d^3}\right]^2} \\
 &\Rightarrow l_d \sim (B - t)^{\frac{1}{2}} e^{-\frac{\lambda t}{2}}, \\
 l_d(t = t_s) &= 0 \rightarrow B = t_s, \quad \lambda = \frac{2}{t_s},
 \end{aligned}$$

$$\begin{aligned}
 \left( \frac{(l'_1)(l_d)^2}{\sqrt{D_{l_d, l_1}}} \right)' &\simeq \frac{1}{\sqrt{D_{l_d, l_1}}} (l'_1 l_d^2) l_d \sim (t_s - t)^{\frac{1}{2}} e^{-\frac{t}{t_s}} \\
 &\Rightarrow D_{l_d, l_1}^{\frac{1}{2}} \sim (t - t_s)^{-3} \\
 &\Rightarrow (l'_1(t_s - t)^4)' \simeq (l'_1(t_s - t)^4) \\
 &\Rightarrow l_1 \sim \frac{1}{(t_s - t)^2} \left( \frac{t_s}{(t_s - t)} - 1 \right) e^{-l_0(t_s - t)}.
 \end{aligned} \quad (16)$$

This equation shows that as time passes, two  $M1$ -branes move toward each other and  $l_d$  decreases while the size of  $M1$  increases. We can show that a wormhole is formed between these  $M1$ -branes, which, by dissolving in them, are the cause of their growth. Before discussing this subject, we will construct  $Mp$ -branes from gluing  $M1$ -branes. To this end, by Eq. (6), we use the following replacements in the action of the branes:

$$\begin{aligned}
 i, j &= a, b \Rightarrow \langle [X^i, X^j, X^k], [X^i, X^j, X^k] \rangle \\
 &\Rightarrow \langle [X^a, X^j, X^i], [X^a, X^j, X^i] \rangle = \frac{1}{2} \langle \partial_a X^i, \partial_a X^i \rangle \sum X_j^2 \\
 i, j &= a, b \Rightarrow \langle [X^i, X^j, X^k], [X^i, X^j, X^k] \rangle \\
 &\Rightarrow \langle [X^a, X^b, X^i], [X^a, X^b, X^i] \rangle = \frac{1}{2} \langle \partial_b \partial_a X^i, \partial_b \partial_a X^i \rangle \\
 i, j, k &= a, b, c \Rightarrow \langle [X^i, X^j, X^k], [X^i, X^j, X^k] \rangle \\
 &\Rightarrow \langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle = \langle F^{abc}, F^{abc} \rangle \\
 Q_{j,k}^i &= \delta_{j,k}^i + i\lambda [X_\alpha^j T^\alpha, X_\beta^k T^\beta, X_\gamma^i T^\gamma] E_{k'jl}^{\alpha, \beta, \gamma} \\
 &\Rightarrow Q_{j,k}^i = \delta_{j,k}^i + i \left( \langle [X^a, X^j, X^i], [X^a, X^j, X^i] \rangle \right. \\
 &\quad \left. + \langle [X^a, X^b, X^i], [X^a, X^b, X^i] \rangle \right. \\
 &\quad \left. + \langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle \right) E_{k'jl}^{\alpha, \beta, \gamma} \\
 &= \delta_{j,k}^i + i \left( \frac{1}{2} \langle \partial_b \partial_a X^i, \partial_b \partial_a X^i \rangle \right.
 \end{aligned}$$

$$+\frac{1}{2}\langle\partial_a X^i, \partial_a X^i\rangle\sum X_j^2+\langle F^{abc}, F^{abc}\rangle E_{k'jl}^{\alpha,\beta,\gamma} \\ \times T_{M1}\int d^2\sigma\Rightarrow T_{Mp}\int d^p\sigma. \quad (17)$$

Applying these relations in Eq. (1), we obtain

$$S=-T_{Mp}\int d^p\sigma\sqrt{-det(O+2\pi l_s^2 G(F))}, \\ G=\sum_{n=0,\dots,p}\frac{1}{n!}\left(-\frac{F}{\beta^2}\right)^n, \\ O=\frac{1}{p}\sum_n(p-n)!\frac{Y^n}{n!}, \\ F=\langle F^{abc}, F^{abc}\rangle, \\ Y=\langle\partial_a X^i, \partial^a X^i\rangle\sum(X^j)^2+\langle\partial_a\partial_b X^i, \partial^a\partial^b X^i\rangle, \\ \beta=\frac{1}{2\pi l_s^2}, \quad (18)$$

where the nonlinear field ( $G$ ) has been introduced in [36–39]. Now, we can show that this action can be reproduced by multiplying the terms of the relevant actions of  $p$   $M1$ 's. For simplicity, we choose  $X^1=\sigma$  and  $X^4=z$ ,  $\sum(X_i)^2\rightarrow F(z)$  where  $z$  is the transverse direction between the branes. Using the action in (18), the Lagrangian for the  $Mp$ -brane can be written as

$$\mathcal{L}=-T_{Mp}\int d\sigma\sigma^p\sqrt{(1+z'^2F(z)+z''^2)+(2\pi l_s^2)^2G(F)} \quad (19)$$

where  $(')$  denotes the derivative with respect to  $\sigma$  and  $z'$  and  $z''$  are the velocity and acceleration of the branes in the transverse dimension. To derive the Hamiltonian, we must obtain the canonical momentum density for the graviton. For simplicity, we will use the method in [40] and [41] and assume that  $F_{001}\neq 0$  and the other components of  $F$  are zero. We get

$$\Pi=\frac{\delta\mathcal{L}}{\delta\partial_t A^{01}}=\frac{\sum_{n=0}^p\frac{n}{n!}\left(-\frac{F}{\beta^2}\right)^{n-1}F_{001}}{\sqrt{(1+z'^2F(z)+z''^2)+(2\pi l_s^2)^2G(F)}}. \quad (20)$$

Thus the Hamiltonian can be written as

$$H=T_{Mp}\int d\sigma\sigma^p\Pi\partial_t A^{01}-\mathcal{L} \\ =4\pi\int d\sigma[\sigma^p\Pi(\partial_t A^{01}-\partial_\sigma A^{00})-\partial_\sigma(\sigma^2\Pi)A_{00}]-\mathcal{L} \quad (21)$$

where we use integration by parts and applied the term proportional to  $\partial_\sigma A^{01}$ . Using the constraint  $(\partial_\sigma(\sigma^p\Pi)=0)$ , we obtain [40]

$$\Pi=\frac{k}{4\pi\sigma^p} \quad (22)$$

where  $k$  is a constant. Replacing  $\Pi$  from the above equation in Eq. (21) gives the following Hamiltonian:

$$H_1=T_{Mp}\int d\sigma\sigma^p \\ \times\sqrt{(1+z'^2F(z)+z''^2)+(2\pi l_s^2)^2\sum_{n=0}^p\frac{n}{n!}\left(-\frac{F}{\beta^2}\right)^n}F_1 \\ F_1=\sqrt{1+\frac{k^2}{\sigma^{2p}}}. \quad (23)$$

For obtaining the explicit form of the wormhole between the branes, we need a Hamiltonian which can be expressed in terms of the separation distance between the branes. To this end, the Lagrangian can be redefined as

$$\mathcal{L}=-T_{Mp}\int d\sigma\sigma^p \\ \times\sqrt{(1+z'^2F(z)+z''^2)+(2\pi l_s^2)^2\sum_{n=0}^p\frac{n}{n!}\left(-\frac{F}{\beta^2}\right)^n}F_1. \quad (24)$$

With the help of this Lagrangian, we repeat our previous calculations. We can obtain

$$\Pi=\frac{\delta\mathcal{L}}{\delta\partial_t A^{01}} \\ =\frac{\sum_n^p\frac{n(n-1)}{n!}\left(-\frac{F}{\beta^2}\right)^{n-1}F_{001}}{\sqrt{(1+z'^2F(z)+z''^2)+(2\pi l_s^2)^2\sum_{n=0}^p\frac{n}{n!}\left(-\frac{F}{\beta^2}\right)^n}}. \quad (25)$$

Therefore the new Hamiltonian can be constructed as

$$H_2=T_{Mp}\int d\sigma\sigma^pF_1\Pi\partial_t A^{01}-\mathcal{L} \\ =\int d\sigma[\sigma^pF_1\Pi(\partial_t A^{01}-\partial_\sigma A^{00})-\partial_\sigma(F_1\sigma^p\Pi)A_{00}]-\mathcal{L} \quad (26)$$

where like in the previous step, we have used in the second step an integration by parts for the term proportional to  $\partial_\sigma A^{01}$  like the method in [40]. Imposing the constraint  $(\partial_\sigma(\sigma^p\Pi)=0)$ , we obtain

$$\Pi=\frac{k}{4F_1\pi\sigma^p}. \quad (27)$$

By replacing the momentum of Eqs. (27) in (26) we derive the following Hamiltonian:

$$\begin{aligned}
H_2 &= T_{Mp} \int d\sigma \sigma^p \\
&\times \sqrt{(1+z'^2 F(z) + z''^2)^p + (2\pi l_s^2)^2 \sum_n \frac{n(n-1)}{n!} \left(-\frac{F}{\beta^2}\right)^n} F_2 \\
F_2 &= F_1 \sqrt{1 + \frac{k^2}{F_1^2 \sigma^{2p}}}
\end{aligned} \quad (28)$$

and by repeating these calculations  $p$  times, we obtain

$$\begin{aligned}
H_p &= T_{Mp} \int d\sigma \sigma^p \\
&\times \sqrt{(1+z'^2 F(z) + z''^2)^p + (2\pi l_s^2)^2 \sum_{n=0}^p \frac{n(n-1) \dots (n-n)}{n!} \left(-\frac{F}{\beta^2}\right)^n} F_p \\
&= T_{Mp} \int d\sigma \sigma^p (\sqrt{1+z'^2 F(z) + z''^2})^p F_{\text{tot}}, \\
F_{\text{tot}} &= \sqrt{1 + \frac{k^2}{F_{p-1}^2 \sigma^{2p}}} \sqrt{1 + \frac{k^2}{F_{p-2}^2 \sigma^{2p}}} \dots \sqrt{1 + \frac{k^2}{F_1^2 \sigma^{2p}}} \sqrt{1 + \frac{k^2}{\sigma^{2p}}}, \\
F_n &= F_{n-1} \sqrt{1 + \frac{k^2}{F_{n-1}^2 \sigma^{2p}}}.
\end{aligned} \quad (29)$$

By growing branes ( $\sigma \rightarrow \infty$ ), the canonical density ( $\Pi$ ) in Eq. (27) becomes small. This is because this momentum is related with the effect of one graviton on the total size of one brane and, consequently, by increasing the length of one brane, this effect decreases. However, by joining M1-branes to each other, the number of gravitons on the branes increases and the total momentum density of gravity, which is the sum over the momentum densities of gravitons is enhanced. Thus, this total momentum becomes large and plays the main role in the evolution of the universe's branes. At this stage, for the case of  $\frac{k}{\sigma^p} \ll 1$ , we can reproduce the Hamiltonian of the  $Mp$ -brane by multiplying the Hamiltonians of the M1's:

$$\begin{aligned}
H_p &= T_{Mp} \int d\sigma \sigma^p (\sqrt{1+z'^2 F(z) + z''^2})^p F_{\text{tot}}, \\
k &= k'^p \rightarrow F_{\text{tot}} \\
&= \sqrt{1 + \frac{k^2}{F_{p-1}^2 \sigma^{2p}}} \sqrt{1 + \frac{k^2}{F_{p-2}^2 \sigma^{2p}}} \dots \\
&\sqrt{1 + \frac{k^2}{F_1^2 \sigma^{2p}}} \sqrt{1 + \frac{k^2}{\sigma^{2p}}} \\
&\simeq \sigma^p \sqrt{1 + \frac{k'^2 p}{\sigma^{2p}} + p \frac{k'^2 p-1}{\sigma^{2p-1}} + \dots} = \sqrt{\left(1 + \frac{k'^2}{\sigma^2}\right)^p} \Rightarrow \\
H_p &= T_{Mp} \int d\sigma \sigma^{2p} (\sqrt{1+z'^2 F(z) + z''^2})^p \left(\sqrt{1 + \frac{k'^2}{\sigma^2}}\right)^p \\
&= T_{Mp} \int d^p \sigma \sigma^p (\sqrt{1+z'^2})^p \left(\sqrt{1 + \frac{k'^2}{\sigma^2}}\right)^p
\end{aligned}$$

$$\begin{aligned}
\Rightarrow H_p &= \left( T_{M1} \int d\sigma \sigma \sqrt{1+z'^2 F(z) + z''^2} \sqrt{1 + \frac{k'^2}{\sigma^2}} \right)^p = H_1^p \\
H_1 &= T_{M1} \int d\sigma \sigma \sqrt{1+z'^2 F(z) + z''^2} \sqrt{1 + \frac{k'^2}{\sigma^2}}
\end{aligned} \quad (30)$$

where we have used of this assumption that  $T_{Mp} = (T_{M1})^p$ . As can be seen from the above equation, each  $Mp$ -brane can be constructed of  $p$  M1-branes. Also, we can show that each M1-brane produces a wormhole. To this end, using the above Hamiltonian and assuming that the acceleration of branes is smaller than the velocity of the branes in the transverse dimension ( $z'' \ll z'$ ) and  $F(z) \sim z^2$ , we derive the following equation of motion  $z$  for any M1:

$$\begin{aligned}
-z'_{M1} &= \left( \frac{V_1(\sigma)^2}{V_1(\sigma_0)^2} - 1 \right)^{-1/2} z, \\
V_1 &= \sigma F_1 = \sigma \sqrt{1 + \frac{k'^2}{\sigma^2}}.
\end{aligned} \quad (31)$$

The solution of this equation is

$$z_{M1} = e^{\int_{\sigma_0}^{\sigma} d\sigma' \left( \frac{V_1(\sigma')^2}{V_1(\sigma_0)^2} - 1 \right)^{-1/2}}. \quad (32)$$

Thus, the separation distance between two branes is

$$\Delta_{M1} = 2z_{M1} = 2e^{\int_{\sigma_0}^{\sigma} d\sigma' \left( \frac{V_1(\sigma')^2}{V_1(\sigma_0)^2} - 1 \right)^{-1/2}} \quad (33)$$

where  $\sigma_0$  is the throat of a wormhole between two M1-branes of two different branes. Thus, each  $Mp$ -brane is constructed from  $p$  M1-branes; each of them produces a wormhole and connects with the M1 of the other branes.

Now, we can derive the relevant action for  $Mp$ -branes by multiplying the action of  $p$  M1-branes by using the antisymmetric form  $\delta$ :

$$\begin{aligned}
S_{Mp} &= -T_{Mp} \int dt \mathcal{L}_{Mp}, \\
\mathcal{L}_{Mp} &= \det(M) \quad L_{M1,i} = L_{a_i}^{b_i} = \det(M_i) \sim M_i \\
&\text{where } \det(M) = \delta_{b_1 b_2 \dots b_n}^{a_1 a_2 \dots a_n} M_{a_1}^{b_1} \dots M_{a_p}^{b_p} \Rightarrow \\
\mathcal{L}_{Mp} &= \det(M) = \delta_{b_1 b_2 \dots b_n}^{a_1 a_2 \dots a_n} L_{a_1}^{b_1} \dots L_{a_p}^{b_p} \\
&\text{where } \delta_{b_1 b_2 \dots b_n}^{a_1 a_2 \dots a_n} \delta_{\mu_1 \nu_1}^{\rho_1 \sigma_1} \dots \delta_{\mu_p \nu_p}^{\rho_p \sigma_p} = \delta_{\mu_1 \nu_1 \dots \mu_p \nu_p}^{\rho_1 \sigma_1 \dots \rho_p \sigma_p}, \\
\sqrt{-g} &= \sqrt{-\det(g)} = \sqrt{-\det(g_1 g_2 \dots g_p)} \\
&= \sqrt{-\det(g_1) \det(g_2) \dots \det(g_p)},
\end{aligned} \quad (34)$$

so we have

$$\begin{aligned}
S_{Mp} &= -(T_{M1})^p \int dt \delta_{b_1 b_2 \dots b_n}^{a_1 a_2 \dots a_n} L_{a_1}^{b_1} \dots L_{a_p}^{b_p} \\
&= -(T_{M1})^p \int dt \int d^p \sigma \delta_{b_1 b_2 \dots b_n}^{a_1 a_2 \dots a_n}
\end{aligned}$$



$$\begin{aligned}
 & \left[ \sqrt{-g_1} \left( \delta_{\mu_1 \nu_1}^{\rho_1 \sigma_1} [\kappa_{\mu_1}^{\rho_1} \kappa_{\nu_1}^{\sigma_1} \sum (X^i)^2 + (\partial_\lambda \kappa_{\mu_1}^{\rho_1} \partial^\lambda \kappa_{\nu_1}^{\sigma_1})] \right. \right. \\
 & \left. \left. - m_g^2 \delta_{\mu_1 \nu_1}^{\rho_1 \sigma_1} (R_{\rho_1 \sigma_1}^{\mu_1 \nu_1} + [\kappa_{\mu_1}^{\rho_1} \kappa_{\nu_1}^{\sigma_1} \sum (X^i)^2 + (\partial_\lambda \kappa_{\mu_1}^{\rho_1} \partial^\lambda \kappa_{\nu_1}^{\sigma_1})]) \right) \right]_{a_1}^{b_1} \\
 & \times \left[ \sqrt{-g_p} \left( \delta_{\mu_p \nu_p}^{\rho_p \sigma_p} [\kappa_{\mu_p}^{\rho_p} \kappa_{\nu_p}^{\sigma_p} \sum (X^i)^2 + (\partial_\lambda \kappa_{\mu_p}^{\rho_p} \partial^\lambda \kappa_{\nu_p}^{\sigma_p})] \right. \right. \\
 & \left. \left. - m_g^2 \delta_{\mu_p \nu_p}^{\rho_p \sigma_p} (R_{\rho_p \sigma_p}^{\mu_p \nu_p} + [\kappa_{\mu_p}^{\rho_p} \kappa_{\nu_p}^{\sigma_p} \sum (X^i)^2 + (\partial_\lambda \kappa_{\mu_p}^{\rho_p} \partial^\lambda \kappa_{\nu_p}^{\sigma_p})]) \right) \right]_{a_p}^{b_p} \\
 & = -(T_{Mp}) \int dt \int d^p \sigma \left[ \sqrt{-g} \left( \sum_{n=1}^p \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} \right. \right. \\
 & \times F(X)^p \kappa_{\rho_1}^{\mu_1} \kappa_{\sigma_1}^{\nu_1} \dots \kappa_{\rho_n}^{\mu_n} \kappa_{\sigma_n}^{\nu_n} - \sum_{n=1}^p m_g^2 \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} R_{\rho_1 \sigma_1}^{\mu_1 \nu_1} \dots R_{\rho_n \sigma_n}^{\mu_n \nu_n} \\
 & - \sum_{n=1}^p m_g^2 F(X)^p \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} \kappa_{\rho_1}^{\mu_1} \partial^{\nu_1} \kappa_{\sigma_1}^{\nu_1} \dots \kappa_{\rho_n}^{\mu_n} \kappa_{\sigma_n}^{\nu_n} \\
 & \left. \left. + \sum_{n=1}^p (1 - m_g^2) \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} \partial_\lambda \kappa_{\rho_1}^{\mu_1} \partial^\lambda \kappa_{\sigma_1}^{\nu_1} \dots \partial_\lambda \kappa_{\rho_n}^{\mu_n} \partial^\lambda \kappa_{\sigma_n}^{\nu_n} + \dots \right) \right]. \quad (35)
 \end{aligned}$$

This action includes all terms in nonlinear gravity theories like Lovelock [42, 43] and massive gravity [29–33]. In addition, some extra terms are predicted only in this model. Now, we calculate the number of degrees freedom on the universe brane and in the bulk. Previously, we showed that 2-form gauge fields are the main cause of the appearance of a wormhole between  $M$ -branes. Thus, the difference between numbers of degrees of freedom on the brane and in the bulk is due to these fields. Using Eqs. (9) and (15) and assuming  $A^{22} \sim g^{22} \sim l_1$  and  $A^{ii} = g_{ij} = 0$ , we get

$$\begin{aligned}
 A^{00} &= g^{00} = -1 \quad A^{22} \sim g^{22} \sim l_1 \quad A^{ij} = 0, i, j \neq 0, 2, \\
 N_{\text{sur}} - N_{\text{bulk}} &= 2(T_{Mp}) \int dt \int d^p \sigma \\
 & \times \left[ \sqrt{-g} \left( \sum_{n=1}^p \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} \langle F^{\rho_1 \sigma_1} \lambda, F^\lambda \mu_1 \nu_1 \rangle \dots \langle F^{\rho_n \sigma_n} \lambda, F^\lambda \mu_n \nu_n \rangle \right) \right] \\
 & = 2(T_{Mp}) \int dt \int d^p \sigma \left[ \sqrt{-g} \left( \sum_{n=1}^p \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} R_{\rho_1 \sigma_1}^{\mu_1 \nu_1} \dots R_{\rho_n \sigma_n}^{\mu_n \nu_n} \right) \right] \\
 & \simeq (T_{Mp}) V \sum_n \left[ \frac{l_0^n}{2^n (2n-1) (t_s - t)^{2n-1}} \left[ 1 - \frac{t_s^{1/2}}{\sqrt{t_s - t}} \right]^{2n} \right. \\
 & \left. - \frac{l_{d,0}^{2n}}{t_s^{2n}} (t_s - t)^n \right] e^{\frac{-2nt}{t_s}} \quad (36)
 \end{aligned}$$

where  $V$  is the volume of brane,  $p = 3$  is related to our universe and the number 2 is related to exchanging a graviton between two branes which produce two sections of a wormhole. We also have used the fact that  $m_g^2 = [(\lambda)^2 \det([X_\alpha^j T^\alpha, X_\beta^k T^\beta, X_\gamma^{k'} T^{\gamma'}])] \sim 1 + l_d^3$ . This equation shows that by approaching branes, the difference between the number of degrees of freedom increases and this causes the growth of branes and the universe's expansion. We also have

$$A^{00} = g^{00} = -1 \quad A^{22} \sim g^{22} \sim l_1 \quad A^{ij} = 0i, j \neq 0, 2,$$

$$\begin{aligned}
 X^2 &= l_1, \quad X^i = 0i \neq 2, \\
 N_{\text{sur}} + N_{\text{bulk}} &= E_{Mp} + E_{\text{anti-M}p} \\
 &= -2(T_{Mp}) \int dt \int d^p \sigma \\
 & \left[ \sqrt{-g} \left( \sum_{n=1}^p \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} F(X)^p \kappa_{\rho_1}^{\mu_1} \kappa_{\sigma_1}^{\nu_1} \dots \kappa_{\rho_n}^{\mu_n} \kappa_{\sigma_n}^{\nu_n} \right. \right. \\
 & - \sum_{n=1}^p m_g^2 \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} R_{\rho_1 \sigma_1}^{\mu_1 \nu_1} \dots R_{\rho_n \sigma_n}^{\mu_n \nu_n} \\
 & - \sum_{n=1}^p m_g^2 F(X)^p \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} \kappa_{\rho_1}^{\mu_1} \kappa_{\sigma_1}^{\nu_1} \dots \kappa_{\rho_n}^{\mu_n} \kappa_{\sigma_n}^{\nu_n} \\
 & \left. \left. + \sum_{n=1}^p (1 - m_g^2) \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} \partial_\lambda \kappa_{\rho_1}^{\mu_1} \partial^\lambda \kappa_{\sigma_1}^{\nu_1} \dots \partial_\lambda \kappa_{\rho_n}^{\mu_n} \partial^\lambda \kappa_{\sigma_n}^{\nu_n} + \dots \right) \right] \\
 & = (T_{Mp}) V \sum_n \left[ \frac{6^n l_{1,+}^{2n}}{(t_s - t)^{8n}} + \frac{l_{1,+}^{2n} l_0^{2n}}{(t_s - t)^{2n}} + \frac{2^n l_{1,+}^{2n}}{(t_s - t)^{6n}} \right] \\
 & \times \left[ 1 - \frac{t_s}{(t_s - t)} \right]^{2n} e^{-2nl_0(t_s - t)} + (T_{Mp}) V \sum_n \left[ \frac{l_{d,0}^{2n}}{t_s^{2n}} (t_s - t)^n \right] e^{\frac{-2nt}{t_s}}. \quad (37)
 \end{aligned}$$

Solving Eqs. (36) and (37), we can obtain the explicit form of number of degrees of freedom of bulk and brane:

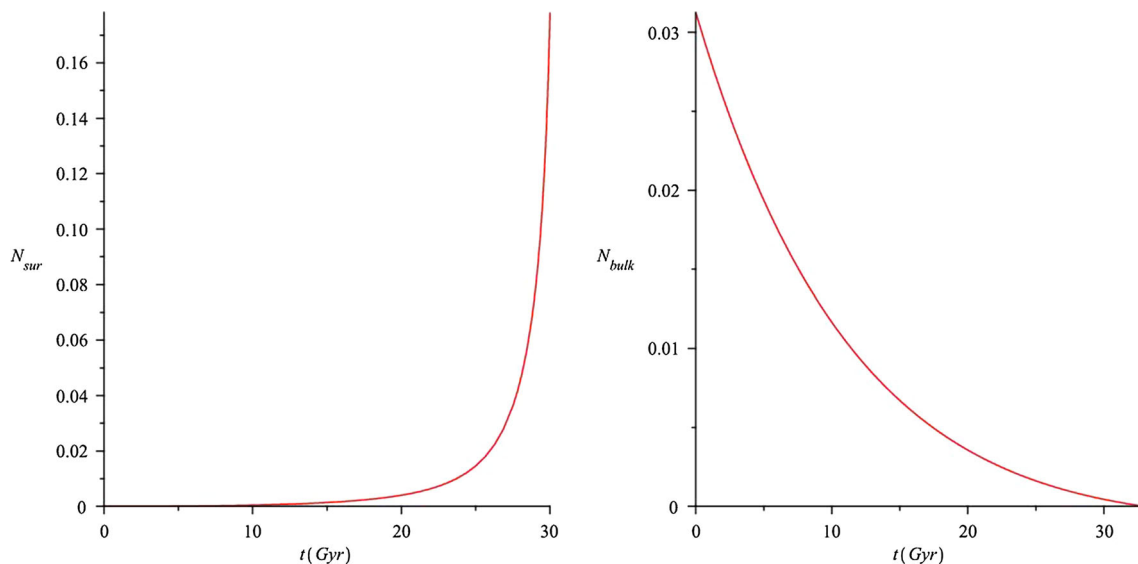
$$\begin{aligned}
 N_{\text{sur}} &= (T_{Mp}) V \sum_{n=1}^p \left[ \frac{6^n l_{1,+}^{2n}}{(t_s - t)^{8n}} + \frac{l_{1,+}^{2n} l_0^{2n}}{(t_s - t)^{2n}} + \frac{2^n l_{1,+}^{2n}}{(t_s - t)^{6n}} \right] \\
 & \times \left[ 1 - \frac{t_s}{(t_s - t)} \right]^{2n} e^{-2nl_0(t_s - t)} + (T_{Mp}) V \\
 & \times \sum_{n=1}^p \left[ \frac{l_0^n}{2^n (2n-1) (t_s - t)^{2n-1}} \left[ 1 - \frac{t_s^{1/2}}{\sqrt{t_s - t}} \right]^{2n} \right] e^{\frac{-2nt}{t_s}}, \quad (38)
 \end{aligned}$$

$$N_{\text{bulk}} = 2(T_{Mp}) V \sum_{n=1}^p \left[ \frac{l_{d,0}^{2n}}{t_s^{2n}} (t_s - t)^n \right] e^{\frac{-2nt}{t_s}}. \quad (39)$$

Clearly, at the beginning ( $t = 0$ ), the number of degrees of freedom on the surface of brane is zero; while, by evolving the time and letting the branes approach toward each other, the number of degrees of freedom on the brane increases and tends to infinity (see Fig. 1 Left). On the other hand, the number of degrees of freedom in the bulk decreases with time and shrinks to zero at the colliding point ( $t = t_s$ ) (see Fig. 1 Right).

### 3 Contraction branch of cosmic space in Padmanabhan model

Until now, we have shown that by letting the branes approach each other, their size grows causing the expansion of the universe. Now, we will show that near the collision point,



**Fig. 1** (Left)  $N_{\text{sur}}$  is increasing from zero at  $t = 0$  to infinity at  $t = t_s$ . (Right)  $N_{\text{bulk}}$  is decreasing from certain value at  $t = 0$  to zero at  $t = t_s$ . We have assumed  $p = 3$  for the 3 + 1-dimensional M3 which our universe is located on it and the time of collision between the branes is  $t_s = 33\text{Gyr}$

branes become compact, the universe contracts, and gravity changes to anti-gravity. This causes the branes to get away from each other. To this end, let us consider Eqs. (13) and (15) near the colliding point:

$$\begin{aligned}
 t \rightarrow t_s &\Rightarrow l_d \sim \frac{l_{d,0}}{t_s^{3/2}} (t_s - t)^{3/2} \rightarrow 0 \\
 &\Rightarrow l_1 \sim \frac{l_{1,+}}{(t_s - t)^2} \left( 1 + \frac{1}{l_{1,0}(t_s - t)} \right) e^{-l_0(t_s - t)} \\
 &\rightarrow l'_1 l_d \sim \frac{1}{(t_s - t)^3} \rightarrow \infty \Rightarrow \\
 D_{l_d, l_1} &= [l_1^2 + (l'_d)^2 + (l''_d)^2 (1 + l_d^2)^{-1}] \left[ 1 - \frac{1}{l_d^3} \right] \\
 &\quad - (l'_1)^2 (l_d)^2 \ll 0. \quad (40)
 \end{aligned}$$

This equation shows that by letting the M1-branes get close to each other,  $D_{l_d, l_1} \ll 0$  and thus the expression under  $\sqrt{\phantom{x}}$  in the action in Eq. (13) becomes negative. This means that the square energy of system becomes negative and some tachyonic states are produced. To solve this problem, M1-branes become compact and the sign of gravity changes. To show this, we use the method in [13, 27, 28] and define  $\langle X^{10} \rangle = \frac{R}{l_p^{3/2}}$  where  $l_p$  is the Planck length. We can write

$$\begin{aligned}
 [X^a, X^b, X^c] &= F^{abc} \quad [X^a, X^b] = F^{ab}, \\
 F_{abc} &= \partial_a A_{bc} - \partial_b A_{ca} + \partial_c A_{ab}, \quad F_{ab} = \partial_a A_b - \partial_b A_a, \\
 \Sigma_{a,b,c=0}^{10} \langle F^{abc}, F_{abc} \rangle &= \Sigma_{a,b,c=0}^{10} \langle [X^a, X^b, X^c], \\
 &\quad [X_a, X_b, X_c] \rangle \\
 &= -\Sigma_{a,b,c,a'b'c'=0}^{10} \varepsilon_{abcD} \varepsilon_{a'b'c'D}^D X^a X^b X^c X_{a'} X_{b'} X_{c'} \\
 &= -6 \Sigma_{a,b,a',b'=0}^9 \varepsilon_{ab10D} \varepsilon_{a'b'10}^D X^a X^b X^{10} X_{a'} X_{b'} X_{10}
 \end{aligned}$$

$$\begin{aligned}
 &-6 \Sigma_{a,b,c,a',b',c'=0,\neq 10}^9 \varepsilon_{abcD} \varepsilon_{a'b'c'D}^D X^a X^b X^c X_{a'} X_{b'} X_{c'} \\
 &= -6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{a,b,a',b'=0}^9 \varepsilon_{ab10D} \varepsilon_{a'b'10}^D X^a X^b X_{a'} X_{b'} \\
 &\quad -6 \Sigma_{a,b,c,a',b',c'=0,\neq 10}^9 \varepsilon_{abcD} \varepsilon_{a'b'c'D}^D X^a X^b X^c X_{a'} X_{b'} X_{c'} \\
 &= -6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{a,b=0}^9 [X^a, X^b]^2 \\
 &\quad -6 \Sigma_{a,b,c,a',b',c'=0,\neq 10}^9 \varepsilon_{abcD} \varepsilon_{a'b'c'D}^D X^a X^b X^c X_{a'} X_{b'} X_{c'} \\
 &= -6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{a,b=0}^9 F^{ab} F_{ab} + E_{\text{Extra}}. \quad (41)
 \end{aligned}$$

This equation shows that the 2-form fields in 11-dimensional space-time transit to a 1-form field due to compactification and the sign of the self energy changes. Using Eq. (41), we can replace all 2-form terms in gravity theories by 1-form terms:

$$\begin{aligned}
 A_b &= e_b \quad F_{ab} = \partial_a e_b - \partial_b e_a \quad \kappa^a b = \partial^a e_b \\
 \Sigma_{\rho,\sigma,\mu,v=0}^{10} R_{\mu\nu}^{\rho\sigma} &= \Sigma_{\rho,\sigma,\mu,v,\lambda=0}^{10} \langle F^{\rho\sigma} \lambda, F^\lambda_{\mu\nu} \rangle \\
 &= \Sigma_{\rho,\sigma,\mu,v,\lambda=0}^{10} \langle [X^\rho, X^\sigma, X_\lambda], [X^\lambda, X_\mu, X_\nu] \rangle \\
 &\Rightarrow -6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{\rho,\sigma,\mu,v=0}^9 [X^\rho, X^\sigma] [X_\mu, X_\nu] \\
 &= -6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{\rho,\sigma,\mu,v=0}^9 F^{\rho\sigma} F_{\mu,\nu} \\
 &= -6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{\rho,\sigma,\mu,v=0}^9 \delta_{\rho'\sigma'}^{\rho\sigma} \delta_{\mu'\nu'}^{\mu\nu} \partial^{\rho'} e^{\sigma'} \partial_{\mu'} e_{\nu'} \\
 &= -6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{\rho,\sigma,\mu,v=0}^9 \delta_{\rho'\sigma'}^{\rho\sigma} \delta_{\mu'\nu'}^{\mu\nu} \kappa^{\rho'\sigma'} \kappa_{\mu'\nu'}
 \end{aligned}$$



$$\Sigma_{\rho,\sigma,\mu,\nu=0}^{10} R = -6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{\rho,\sigma,\mu,\nu=0}^9 \delta_{\rho'\sigma'}^{\mu\nu} \delta_{\mu\nu}^{\mu'\nu'} \kappa_{\rho'\sigma'}^{\rho'\sigma'} \kappa_{\mu'\nu'}^{\mu'\nu'}. \quad (42)$$

With the help of these relations, we can show that the sign of Lovelock gravity changes:

$$\begin{aligned} \sum_{n=1}^p m_g^{2n} \delta_{\mu_1\nu_1\dots\mu_n\nu_n}^{\rho_1\sigma_1\dots\rho_n\sigma_n} R_{\rho_1\sigma_1}^{\mu_1\nu_1} \dots R_{\rho_n\sigma_n}^{\mu_n\nu_n} \\ \Rightarrow - \sum_{n=1}^p m_g^{2n} \left( \frac{R^{2n}}{l_p^{3n}} \right) \delta_{\mu_1\nu_1\dots\mu_n\nu_n}^{\rho_1\sigma_1\dots\rho_n\sigma_n} \kappa_{\rho_1}^{\mu_1} \kappa_{\sigma_1}^{\nu_1} \dots \kappa_{\rho_n}^{\mu_n} \kappa_{\sigma_n}^{\nu_n}. \end{aligned} \quad (43)$$

This equation shows that by compactifying the Mp-brane, nonlinear theories like Lovelock gravity convert to other types of nonlinear gravity theories with opposite sign. This means that by compactifying the branes, gravity changes to anti-gravity. We can study other effects of compactifications of Mp-brane by extending the relations in (43):

$$\begin{aligned} \partial_a X^i \partial_a X^i \sum (X^j)^2 &= \Sigma_{a,i,j=0}^{10} \langle [X^a, X^i, X^j], [X_a, X_i, X_j] \rangle \\ &\Rightarrow -6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{i,j=0}^9 [X^a, X^j] [X_a, X_j] \\ &= -6 \left( \frac{R^2}{l_p^3} \right) \partial_a X^i \partial_a X^i \Rightarrow F(X) = \sum (X^j)^2 = 1 \\ \partial_a \partial_b X^i \partial_a \partial_b X^i &= \Sigma_{a,b,i=0}^{10} \langle [X^a, X^b, X^i], [X_a, X_b, X_i] \rangle \\ &\Rightarrow -6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{i,b=0}^9 [X^b, X^i] [X_b, X_i] = -6 \left( \frac{R^2}{l_p^3} \right) \partial_a X^i \partial_a X^i. \end{aligned} \quad (44)$$

When scalars attached to branes give the index of the branes, they play the role of graviton. Under these conditions, using Eq. (44), we can obtain the following relations:

$$\begin{aligned} X^i \rightarrow e^c &\Rightarrow \partial_a \partial_b X^i \rightarrow \partial_a \kappa_b^c \\ \partial_a \kappa_b^c \partial_a \kappa_b^c &\rightarrow \partial_a \partial_b X^i \partial_a \partial_b X^i \\ &= \Sigma_{a,b,i=0}^{10} \langle [X^a, X^b, X^i], [X_a, X_b, X_i] \rangle \\ &\Rightarrow -6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{i,b=0}^9 [X^b, X^i] [X_b, X_i] \\ &= -6 \left( \frac{R^2}{l_p^3} \right) \partial_a X^i \partial_a X^i \rightarrow -6 \left( \frac{R^2}{l_p^3} \right) \kappa_b^c \kappa_b^c \end{aligned}$$

$$\begin{aligned} &\Rightarrow \sum_{n=1}^p \delta_{\mu_1\nu_1\dots\mu_n\nu_n}^{\rho_1\sigma_1\dots\rho_n\sigma_n} \partial_\lambda \kappa_{\rho_1}^{\mu_1} \partial^\lambda \kappa_{\sigma_1}^{\nu_1} \dots \partial_\lambda \kappa_{\rho_n}^{\mu_n} \partial^\lambda \kappa_{\sigma_n}^{\nu_n} \\ &\rightarrow -6 \left( \frac{R^2}{l_p^3} \right) \Sigma_{i,b=0}^9 \delta_{\mu_1\nu_1\dots\mu_n\nu_n}^{\rho_1\sigma_1\dots\rho_n\sigma_n} \kappa_{\rho_1}^{\mu_1} \kappa_{\sigma_1}^{\nu_1} \dots \kappa_{\rho_n}^{\mu_n} \kappa_{\sigma_n}^{\nu_n}. \end{aligned} \quad (45)$$

Substituting Eqs. (43) and (45) into the action (37), we observe that four nonlinear terms out of five will be removed by each other and only one term remains:

$$\begin{aligned} S_{Mp} &= -(T_{Mp}) \int dt \int d^p \sigma \\ &\times \left[ \sqrt{-g} \left( \sum_{n=1}^p m_g^{2n} \left( 1 + 6^n \left( \frac{R^{2n}}{l_p^{3n}} \right) \right) \right. \right. \\ &\quad \left. \left. \delta_{\mu_1\nu_1\dots\mu_n\nu_n}^{\rho_1\sigma_1\dots\rho_n\sigma_n} \kappa_{\rho_1}^{\mu_1} \kappa_{\sigma_1}^{\nu_1} \dots \kappa_{\rho_n}^{\mu_n} \kappa_{\sigma_n}^{\nu_n} \right) \right] \end{aligned} \quad (46)$$

where  $m_g^2 = [(\lambda)^2 \det([X^j, X^k])]$  is the square of the graviton mass. This equation shows that compactifying Mp-branes gives rise to anti-gravity. For example, we have

$$\sqrt{-g} R \rightarrow -m_g^2 \sqrt{-g} R \quad (47)$$

for general relativity. In fact, by letting the branes approach each other, they become compact, the universe contracts and gravity changes to anti-gravity. Under these conditions, the branes are getting away from each other and the contraction branch ends. To show this, similar to the previous section, we consider the action of M1-branes and then extend it to higher dimensional compact branes. We can rewrite the action of compacted M1-brane as [13, 17–28]

$$\begin{aligned} S &= -T_{M1} \int d^2 \sigma \, STr \\ &\times \sqrt{-\det(P_{ab}[E_{mn} + E_{mi}(Q^{-1} - \delta)^{ij} E_{jn}] + \lambda F_{ab}) \det(Q_j^i)} \end{aligned} \quad (48)$$

where

$$\begin{aligned} E_{mn} &= G_{mn} + B_{mn}, \quad Q_j^i = \delta_j^i + i\lambda [X^j, X^k] E_{kj}, \\ F_{ab} &= \partial_a A_b - \partial_b A_a \end{aligned} \quad (49)$$

where  $\lambda = 2\pi l_s^2$ ,  $G_{mn} = g_{mn} + \partial_m X^i \partial_n X^i$  and  $X^i$  are scalar fields with mass dimension. Using the above action and assuming that, as in the previous section, the separation distance between two M1 is  $l_d$  and the length of each M1 is  $l_1$ , we can derive the relevant action for the interaction of an M1 with an anti-M1-brane:

$$\begin{aligned} A^a \rightarrow l_1 \quad X^2 \rightarrow l_d \quad X^0 = t \quad X^i = 0, i \neq 0, 2 \\ S &= -T_{M1} \int d^2 \sigma \sqrt{l_d^2 + 1} \sqrt{[l_1^2 + (l'_d)^2] \left[ 1 - \frac{1}{l_d^2} \right] + (l'_1)^2} \\ &= -T_{M1} \int d^2 \sigma V(l_d) \sqrt{D_{l_d, l_1}}, \end{aligned}$$

$$V(l_d) = \sqrt{l_d^2 + 1},$$

$$D_{l_d, l_1} = [l_1^2 + (l'_d)^2] \left[ 1 - \frac{1}{l_d^2} \right] + (l'_1)^2, \quad (50)$$

where the prime denotes the derivative with respect to time. The equations of motion extracted from the above action are

$$\left( \frac{(l'_d)[1 - \frac{1}{l_d^2}]}{\sqrt{D_{l_d, l_1}}} \right)' = \frac{1}{\sqrt{D_{l_d, l_1}}} \\ \times \left( 2l_d^{-3}[(l'_d)^2 + l_1^2]l'_d + \frac{V'}{V}[D_{l_d, l_1} - (l'_d)[1 - \frac{1}{l_d^2}]] \right),$$

$$\left( \frac{2(l'_1)}{\sqrt{D_{l_d, l_1}}} \right)' = \frac{1}{\sqrt{D_{l_d, l_1}}} \left( l_1 \left[ 1 - \frac{1}{l_d^2} \right] - \frac{V'}{V}[l'_1 l'_d] \right). \quad (51)$$

The approximate solutions of the above equations are

$$l_1 \sim \frac{l_{1,-}}{(t - t_s)^2} \left( 1 + \frac{t_s}{(t - t_s)} \right) e^{l_0(t_s - t)}$$

$$l_d \sim \frac{l_{d,-}}{t_s} (t - t_s) \left[ 1 + 2 \ln \left[ 1 + \frac{(t - t_s)}{t_s} \right] \right]. \quad (52)$$

It is clear that at  $t = t_s$ , the separation distance between the branes ( $l_d = 0$ ) is zero and the length of  $M1$  is approximately infinite; while, as time passes, the distance between the  $M1$ -branes increases and the length of the branes decreases. We can examine the correctness of these solutions near the colliding point where branes are very close to each other. In this case, the sizes of the branes before and after collision are approximately equal ( $l_{1,before}(t = t_s) = l_{1,after}(t = t_s)$ ). Using this assumption, the equations in (52) reduce to the following equations:

$$l_d \sim 0 \quad l'_d \sim 0$$

$$\Rightarrow \frac{V'}{V} D_{l_d, l_1} \sim \frac{l_d l'_d}{l_d^2 + 1} \left[ [l_1^2 + (l'_d)^2] \left[ 1 - \frac{1}{l_d^2} \right] + (l'_1)^2 \right]$$

$$\sim -2l_d^{-3}[(l'_d)^2 + l_1^2]l'_d + \frac{l'_d l_1^2}{l_d}$$

and

$$l_{1,before}(t \rightarrow t_s) = l_{1,after}(t \rightarrow t_s) \simeq \left| \frac{t_s}{(t - t_s)^3} \right|$$

$$\Rightarrow l_1^2 + (l'_1)^2 = (l_1)^2 \left[ 1 + \frac{(t - t_s)^2}{t_s^2} \right]$$

$$= (l_1)^2 \left[ \left[ 1 + \frac{(t - t_s)}{t_s} \right]^2 - 2 \frac{(t - t_s)}{t_s} \right]$$

$$\Rightarrow \left( \frac{(l'_d)[1 - \frac{1}{l_d^2}]}{\sqrt{D_{l_d, l_1}}} \right)' = \frac{2}{1 + \frac{(t - t_s)}{t_s}}$$

$$\Rightarrow l_d \sim \frac{l_{d,-}}{t_s} (t - t_s) \left[ 1 + 2 \ln \left[ 1 + \frac{(t - t_s)}{t_s} \right] \right]$$

$$\left( \frac{2(l'_1)}{\sqrt{D_{l_d, l_1}}} \right)' \simeq \frac{2(l'_1)}{\sqrt{D_{l_d, l_1}}}$$

$$\Rightarrow l_1 \sim \frac{l_{1,-}}{(t - t_s)^2} \left( 1 + \frac{t_s}{(t - t_s)} \right) e^{l_0(t_s - t)}. \quad (53)$$

These results can be extended to higher dimensional branes. We have constructed the action of (46) from compactifying terms in the action (35). On the other hand, in the previous section, we have proved that each  $Mp$ -branes can be built from  $p$   $M1$ -branes. We have

$$S_{Mp} = -T_{Mp} \int dt \delta_{b_1 b_2 \dots b_n}^{a_1 a_2 \dots a_n} L_{a_1}^{b_1} \dots L_{a_p}^{b_p} \quad H \sim H_1^p. \quad (54)$$

This means that the results of Eq. (52) can be generalized to  $Mp$ -branes and we can choose the same lengths for all dimensions of the brane:

$$l_2 = \dots l_p = l_1 \sim \frac{l_{1,-}}{(t - t_s)^2} \left( 1 + \frac{t_s}{(t - t_s)} \right) e^{l_0(t_s - t)}. \quad (55)$$

At this stage, we can write the relations between the number of degrees of freedom on the brane and in the bulk and the energy of the system. Until now, we have shown that 1-form gauge fields produce anti-gravity which are the main cause of the inequality between the number of degrees of freedom on the brane and in the bulk. Substituting Eqs. (43), (45), and (46) in (36) and (37), we obtain

$$N_{\text{sur}} - N_{\text{bulk}} = 2(T_{Mp}) \int dt \int d^p \sigma$$

$$\times \left[ \sqrt{-g} \left( \sum_{n=1}^p \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} F^{\rho_1 \sigma_1} F_{\mu_1 \nu_1} \dots F^{\rho_n \sigma_n} F_{\mu_n \nu_n} \right) \right]$$

$$= 2(T_{Mp}) \int dt \int d^p \sigma$$

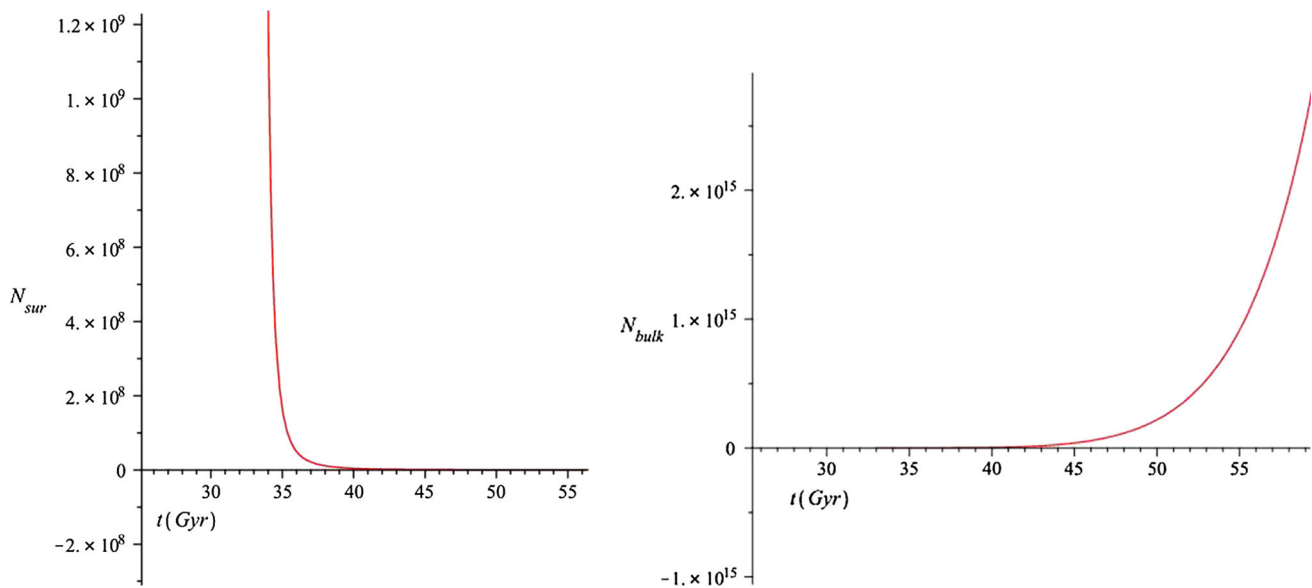
$$\times \left[ \sqrt{-g} \left( \sum_{n=1}^p \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} \kappa_{\rho_1}^{\mu_1} \kappa_{\sigma_1}^{\nu_1} \dots \kappa_{\rho_n}^{\mu_n} \kappa_{\sigma_n}^{\nu_n} \right) \right], \quad (56)$$

$$N_{\text{sur}} + N_{\text{bulk}} = E_{\text{compact-M}3} + E_{\text{compact-anti-M}3}$$

$$= 2(T_{Mp}) \int dt \int d^p \sigma \left[ \sqrt{-g} \left( \sum_{n=1}^p m_g^{2n} (1 + 6^n \left( \frac{R^{2n}}{l_p^{3n}} \right)) \right. \right.$$

$$\left. \left. \times \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} \kappa_{\rho_1}^{\mu_1} \kappa_{\sigma_1}^{\nu_1} \dots \kappa_{\rho_n}^{\mu_n} \kappa_{\sigma_n}^{\nu_n} \right) \right]. \quad (57)$$

Solving the above equations, using Eq. (52) and assuming  $m_g^2 = [(\lambda)^2 \det([X^j, X^k])] = 1 + l_d^2$  and  $R = \frac{l_p^{\frac{3}{2}}}{6}$  we obtain the surface degrees of freedom and the one of the bulk as follows:



**Fig. 2** (Left)  $N_{sur}$  is decreasing from large value at  $t = t_s$  to zero at large time. (Right)  $N_{bulk}$  is increasing from zero at  $t = t_s$  large value for large time. We have assumed  $p = 3$  for the  $3 + 1$ -dimensional M3 which our universe is located on it and time of collision between branes  $t_s = 33$  Gyr

$$\begin{aligned}
 A^a &\rightarrow l_1 \quad X^2 \rightarrow l_d \quad X^0 = t \quad X^i = 0, i \neq 0, 2, \\
 N_{sur} &= (T_{Mp}) \int dt \int d^p \sigma \\
 &\times \left[ \sqrt{-g} \left( \sum_{n=1}^p (l_d^{2n} (1 + 6^n (\frac{R^{2n}}{l_p^{3n}})) + 2) \right. \right. \\
 &\quad \left. \left. \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} \kappa_{\rho_1}^{\mu_1} \kappa_{\sigma_1}^{\nu_1} \dots \kappa_{\rho_n}^{\mu_n} \kappa_{\sigma_n}^{\nu_n} \right) \right] \\
 &\sim (T_{Mp}) V \sum_{n=1}^p \left[ \frac{l_{d,-}^{2n}}{t_s^{2n} (2n-1)} \frac{1}{(t-t_s)^{2n-1}} + \frac{l_{d,-}^{2n}}{t_s^{2n}} t_s (t-t_s) \right. \\
 &\quad \times \ln \left[ 1 + \frac{(t-t_s)}{t_s} \right] \left[ 1 + 2 \ln \left[ 1 + \frac{(t-t_s)}{t_s} \right] \right]^{2n-1} \Big] \\
 &+ (T_{Mp}) V \sum_{n=1}^p \left[ \frac{l_{d,-}^{2n}}{t_s^{2n} (2n+1)} (t-t_s)^{2n+1} \right] \left[ 1 + (t-t_s) \right. \\
 &\quad \times \ln \left[ 1 + \frac{(t-t_s)}{t_s} \right] + (t-t_s) + \frac{t_s^2}{t-t_s} \Big]^n, \quad (58)
 \end{aligned}$$

$$\begin{aligned}
 N_{bulk} &= 2(T_{Mp}) \int dt \int d^p \sigma \\
 &\times \left[ \sqrt{-g} \left( \sum_{n=1}^p (l_d^{2n}) \delta_{\mu_1 \nu_1 \dots \mu_n \nu_n}^{\rho_1 \sigma_1 \dots \rho_n \sigma_n} \kappa_{\rho_1}^{\mu_1} \kappa_{\sigma_1}^{\nu_1} \dots \kappa_{\rho_n}^{\mu_n} \kappa_{\sigma_n}^{\nu_n} \right) \right] \\
 &\sim (T_{Mp}) V \sum_{n=1}^p \left[ \frac{l_{d,-}^{2n}}{t_s^{2n} (2n+1)} (t-t_s)^{2n+1} \right] \\
 &\quad \times \left[ 1 + (t-t_s) \ln \left[ 1 + \frac{(t-t_s)}{t_s} \right] + (t-t_s) + \frac{t_s^2}{t-t_s} \right]^n. \quad (59)
 \end{aligned}$$

These equations show that at the colliding point ( $t = t_s$ ),  $m_g^2 = 1$ , the number of degrees of freedom in the bulk is zero ( $N_{bulk} = 0$ ) and the number of degrees of freedom on the brane surface becomes infinite ( $N_{sur} = \infty$ ). However, as time passes, the number of degrees of freedom on the brane surface decreases and shrinks to zero, while the number of degrees of freedom in the bulk increases (see Fig. 2).

## 4 Summary and discussion

In this paper, we have investigated the Padmanabhan proposal in a system of oscillating branes. In this model, first, a pair of  $M1$ -anti- $M1$ -branes are constructed from joining  $M0$ -branes. During the processes of the formation of these branes, two types of fields emerge. The first type is a scalar field which moves in transverse direction and when glues to branes, plays the role of a graviton scalar mode. The second type lives on the brane, plays the role of graviton tensor modes and causes the formation of a wormhole between the branes. By moving two branes close toward each other, the wormhole dissolves in them and leads to an inequality between the number of degrees of freedom on the surface of the branes and in the bulk. Near the colliding point, the square of the energy of the system becomes negative and for solving this problem, the  $M1$ -branes become compact, 2-form gauge fields convert to 1-form gravity with opposite sign and anti-gravity emerges. Under these conditions, the branes get away from each other and their size decreases. By joining  $M1$ -branes, higher dimensional branes like  $M3$ -branes are produced which become compact and open like

the initial  $M1$ 's. Our universe is located on one of these  $M3$ -branes and, by compactifying them, contracts and by opening, expands. By the expanding universe, the number of degrees of freedom on the surface increases, while the one in the bulk decreases. However, by a contracting universe, the number of degrees of freedom on the surface decreases and the one in bulk is enhanced. In a forthcoming paper, possible observational signatures of this dynamics will be discussed.

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